

**Box 4.10** Extreme value probability distributions

The following extreme value probability distributions (see Equations 4.107 to 4.110) are commonly used to fit the long-term distributions of wave height, water levels etc.

$$\text{Gumbel} \quad P(X) = P(\underline{X} \leq X) = \exp[-\exp(-aX + b)] \quad (4.107)$$

$$\text{Weibull} \quad P(X) = P(\underline{X} \leq X) = 1 - \exp\left[-\left(\frac{X-a}{b}\right)^c\right] \quad (4.108)$$

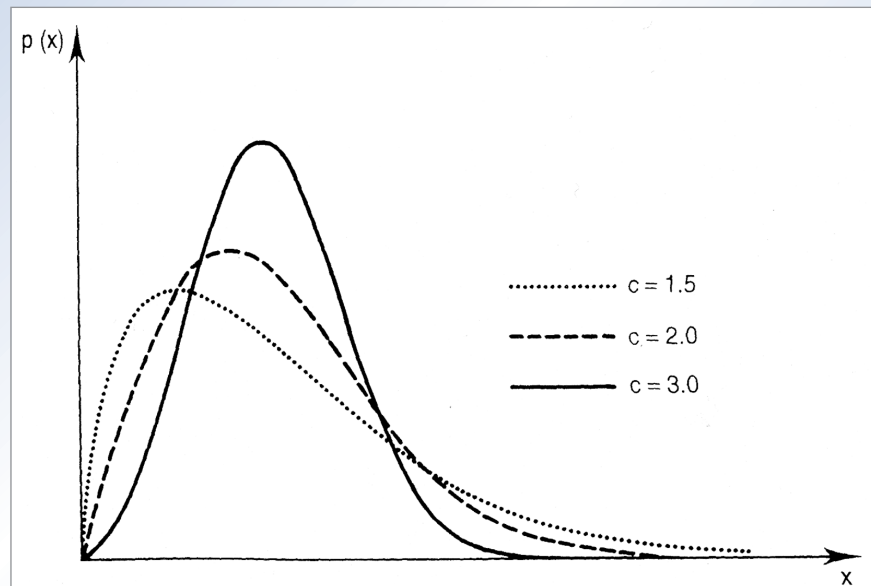
$$\text{Log-normal} \quad p(x) = \frac{1}{a x \sqrt{\pi}} \exp\left[-\left(\frac{\ln(x)-b}{a}\right)^2\right] \quad (4.109)$$

$$\text{Exponential} \quad P(X) = P(\underline{X} \leq X) = 1 - \exp\left[-\frac{X-a}{b}\right] \quad (4.110)$$

where  $P(X)$  is the cumulative probability function, ie the probability that  $\underline{X}$  will not exceed  $X$ , ie  $P(\underline{X} \leq X)$ , and  $p(x)$  is the probability density function of  $x$  and  $p(x) = dP/dx$ .

Note that the three-parameter Weibull distribution reduces to the shifted Rayleigh distribution if  $c = 2$  and  $a \neq 0$  and to the classical shifted Rayleigh distribution if  $c = 2$  and  $a = 0$  (see Figure 4.45).

With  $c = 1$ , the three-parameter Weibull corresponds to the exponential distribution, very often used for extreme wave climate analysis. The more universal nature of the Weibull distribution means that this is often the preferred model.



**Note:** The curve with  $c = 2$  corresponds to the Rayleigh distribution presented in Section 4.2.4.4.

**Figure 4.45** The two-parameter Weibull distribution (third parameter  $a = 0$ )