A shortcoming of the Rayleigh distribution is that it is not bounded by an upper maximum value. Thus the maximum wave height can neither be defined nor computed in a deterministic way from this distribution. However, the representative wave heights $H_{P\%}$ and $H_{1/Q}$ can be computed analytically (see Equations 4.53 and 4.54) from the Rayleigh distribution (eg Massel, 1996; Goda, 2000).

$$\frac{H_{P\%}}{H_{rms}} = \sqrt{-\ln\left(P/100\right)} \tag{4.53}$$

$$\frac{H_{1/Q}}{H_{rms}} = \frac{\sqrt{\pi}}{2} Q\, erfc\left(\sqrt{\ln Q}\right) + \sqrt{\ln Q}, \quad \text{with } \operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} \exp\left(-t^2\right) \mathrm{d}t \tag{4.54}$$

The most important and useful results are listed in Table 4.8. An important issue is the estimation of the maximum value of the wave height for the case of sea-states of finite duration. This maximum wave height cannot be determined in a deterministic manner. One can, however, derive a probability density function for the (statistical) ratio H_{max}/H_s (eg Massel, 1996; Goda, 2000). Two important representative values, namely the mode and the mean values, can be expressed analytically (see Equations 4.55 and 4.56) and computed (see Table 4.9 for some typical results).

Table 4.8 ***Characteristic wave height ratios for a sea-state with a Rayleigh distribution of wave heights***

Characteristic height H	Wave height ratios			
	$H/\sqrt{m_0}$	H/H_m	H/H_{rms}	H/H_s
Standard deviation of free surface $\sigma_\eta = \sqrt{m_0}$	1	0.399	0.353	0.250
Mean wave height H_m	2.507	1	0.886	0.626
Root-mean-square wave height H_{rms}	2.828	1.128	1	0.706
Significant wave height $H_s = H_{1/3}$	4.004	1.597	1.416	1
Wave height $H_{1/10}$	5.090	2.031	1.800	1.273
Wave height $H_{1/100}$	6.673	2.662	2.359	1.668
Wave height $H_{2\%}$	5.594	2.232	1.978	1.397

- ***Mode of the distribution***

The most probable value of the ratio H_{max}/H_s for a record consisting of N waves is given by Equation 4.55.

$$\left[\frac{H_{max}}{H_s}\right]_{mode} \approx \sqrt{\frac{\ln N}{2}} \tag{4.55}$$

- ***Mean value of the distribution***

The mean value of the ratio H_{max}/H_s for a record consisting of N waves (see Equation 4.56). The mean value is greater than the mode, because of the skewed shape of the distribution:

$$\left[\frac{H_{max}}{H_s}\right]_{mean} \approx \left(\sqrt{\frac{\ln N}{2}} + \frac{\gamma}{2\sqrt{2\ln N}}\right) \tag{4.56}$$

where γ = Euler constant ≈ 0.5772.