

**Distribution of individual wave heights in a sea-state**

During each sea-state a (short-term) distribution of wave heights applies. Once the distribution function of wave heights is known, all the characteristic wave heights listed in Table 4.7 can be computed. Some basic and important results for wave distributions are summarised below: first for the deep-water case, and then for the shallow-water case. The latter is more important for the design of coastal structures, but also more difficult to model and parameterise.

- *Distribution of deep-water wave heights*

In deep water the water surface elevation usually follows a Gaussian process and thus the individual wave heights closely follow the Rayleigh distribution. Note that the Rayleigh distribution is a particular case of the Weibull distribution, with a fixed shape parameter of 2 (see Box 4.10). This distribution is fully defined by a single parameter, which may be either the mean wave height  $H_m$  or the root mean square (rms) wave height  $H_{rms}$ , or alternatively the variance of the free-surface elevation  $m_0$ . Equation 4.50 gives the equivalent forms of the cumulative distribution function.

$$P(H) = P(\underline{H} < H) = 1 - \exp\left(-\frac{H^2}{8m_0}\right) = 1 - \exp\left(-\frac{\pi}{4}\left(\frac{H}{H_m}\right)^2\right) = 1 - \exp\left(-\left(\frac{H}{H_{rms}}\right)^2\right) \quad (4.50)$$

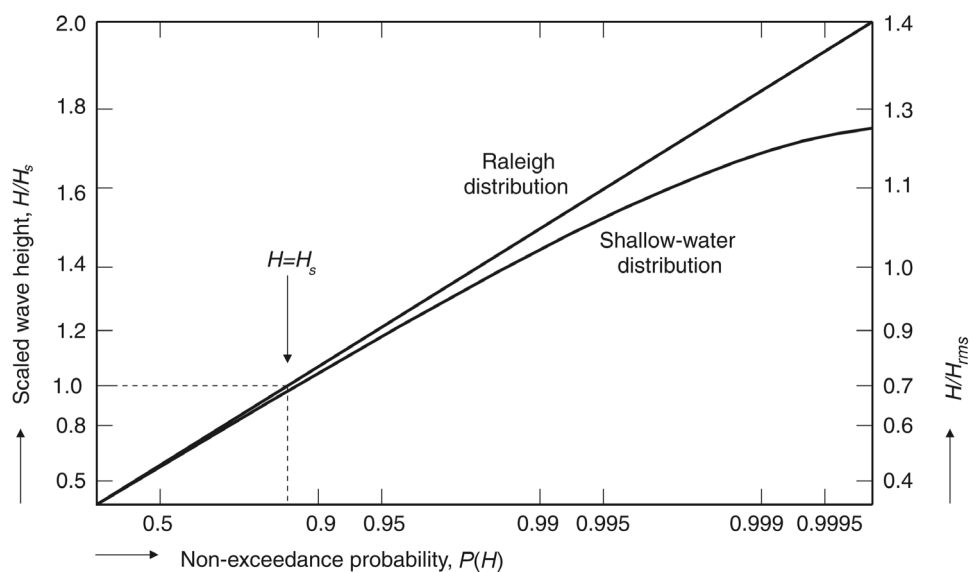
Equation 4.51 gives the corresponding probability density function.

$$p(H) = \frac{H}{4m_0} \exp\left(-\frac{H^2}{8m_0}\right) = \frac{\pi}{2} \frac{H}{H_m^2} \exp\left(-\frac{\pi}{4}\left(\frac{H}{H_m}\right)^2\right) = \frac{2H}{H_{rms}^2} \exp\left(-\left(\frac{H}{H_{rms}}\right)^2\right) \quad (4.51)$$

The variance  $m_0$  can be computed from the free-surface elevation signal  $\eta(t)$  (see Equation 4.52) or from the wave spectrum  $E(f)$  (it corresponds to the area between spectrum and the  $x$ -axis, see Section 4.2.4.5).

$$m_0 = \eta_{rms}^2 = \frac{1}{T} \int_0^T (\eta(t) - \bar{\eta})^2 dt \quad (4.52)$$

Figure 4.27 shows the Rayleigh distribution.



**Figure 4.27** Example of a shallow-water observed distribution of wave heights compared with the Rayleigh distribution