equation accurately whenever necessary, but explicit approximations, such as given in Box 4.3, can also be used.

The propagation velocity of wave crests (**phase speed**) is $c = L/T = \omega/k$ (m/s) and the propagation velocity of energy (**group velocity**) is given by $c_g = \partial\omega/\partial k$ (m/s). In linear wave theory, based on Equation 4.38, the expressions for phase and group velocity are given by Equations 4.39 and 4.40 respectively.

$$c = \frac{g}{\omega}\tanh(kh) = \sqrt{\frac{g}{k}\tanh(kh)} \tag{4.39}$$

$$c_g = nc \quad \text{with} \quad n = \frac{1}{2}\left(1 + \frac{2kh}{\sinh(2kh)}\right) \tag{4.40}$$

Note that the factor n has two asymptotic values: (1) when the relative water depth, kh (-), is small, n tends towards 1; (2) when kh is large n tends towards ½; in this case the wave energy propagates at a speed that is half of that of individual waves. For these asymptotic cases, particular expressions of k, L, c and c_g may be derived analytically and are listed in Table 4.6, together with the non-dimensional criteria for using these approximations. For values in deep water (large value of kh), the subscript "0" or "o" was used conventionally (eg L_o for the deep-water wavelength). Here the latter, "o" (of offshore), is used. From Table 4.6 it should be noted, for example, that in shallow-water conditions, c and c_g do not depend any more on the wave period, T, and that all waves have the same velocity (non-dispersive waves), which, in this case, equals the velocity of energy.

Box 4.3 *Explicit approximations of the linear dispersion relation for water waves*

There are numerous approximations of the dispersion relation given by Equation 4.38. Equation 4.41 gives the rational one proposed by Hunt (1979) at order 9, which is very accurate (always less than 0.01 per cent of relative error in kh):

$$(kh)^2 = (k_o h)^2 + \frac{k_o h}{1 + \sum_{n=1}^{9} a_n (k_o h)^n} \tag{4.41}$$

where $k_o = 2\pi/L_o = \omega^2/g$ = deep-water wave number (rad/m) and the values of a_n are as follows:

$a_1 = 0.66667$ $a_2 = 0.35550$ $a_3 = 0.16084$ $a_4 = 0.06320$ $a_5 = 0.02174$
$a_6 = 0.00654$ $a_7 = 0.00171$ $a_8 = 0.00039$ $a_9 = 0.00011$.

Hunt (1979) also provides a similar formula at order 6, with a relative error in kh always less than 0.2 per cent.

Alternatively, the simpler explicit formulation by Fenton and McKee (1990) (see Equation 4.42) can be used. Although it is less accurate than the former (1.5 per cent of maximum relative error), it is easier to use on a calculator.

$$k = \frac{\omega^2}{g}\left\{\coth\left[\left(\omega\sqrt{\frac{h}{g}}\right)^{3/2}\right]\right\}^{2/3} \quad \text{or equivalent:} \quad L = L_o\left\{\tanh\left[(k_o h)^{3/4}\right]\right\}^{2/3} \tag{4.42}$$

Other explicit expressions have been proposed by Eckart (1952), Wu and Thornton (1986), Guo (2002).